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A BEM FORMULATION FOR COMPUTATIONAL DESIGN OF GROUNDING SYSTEMS IN STRATIFIED SOILS

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Abstract. *Substation grounding design involves computing the equivalent resistance of the earthing system —for reasons of equipment protection—, as well as distribution of potentials on the earth surface —for reasons of human security— when fault conditions occur*[1].
While very crude approximations were available in the sixties, several methods have been proposed in the last three decades, most of them on the basis of practice and intuitive ideas[1,2]. *Although these techniques represented a significant improvement in the area of grounding analysis, a number of problems, such as the computational requirements or the error uncertainty, were reported*[3].
Recently, the authors have identified these widespread intuitive methods as particular cases of a general Boundary Element numerical approach[4]. *Furthermore, starting from this BE formulation it has been possible to develop others more efficient and accurate*[5].
The Boundary Element formulations derived up to this moment are based on the hypothesis —widely assumed in most of the practical techniques and procedures— that the soil can be considered homogeneous and isotropic[5].
A more general BE approach for the numerical analysis of substation grounding systems in nonuniform soils is presented in this paper. The formulation is specially derived for two-layer soil models, widely considered as adequate for most practical cases. The feasibility of this BEM approach is demonstrated by solving two real application problems, in which accurate results for the equivalent resistance and the potential distribution on the ground surface are obtained with acceptable computing requirements.

1. INTRODUCTION

The main objectives of a grounding system are to grant the integrity of the equipment and to ensure the continuity of the electrical supply, providing means to carry and dissipate electric currents into the ground, and to safeguard that a person in the vicinity of grounded installations is not exposed to the danger of suffering a critical electric shock. For the attainment of these aims, the equivalent electrical resistance of the system must be low enough to assure that fault currents dissipate mainly through the grounding grid into the earth, while maximum potential gradients between points that can be contacted by the human body must be kept under certain safe limits (step, touch and mesh voltages)[1].

In the last threee decades, several procedures and methods for substation grounding design and computation have been proposed. Most of them are founded on practice, on semiempirical works or on the basis of intuitive ideas, such as superposition of punctual current sources and error averaging[2]. Although these techniques represented a significant improvement in the area of earthing analysis, a number of problems have been reported: applicability limited to very simple grounding arrangements of electrodes in uniform soils, large computational requirements, unrealistic results when discretization of conductors is increased, and uncertainty in the margin of error[3].

In the last years a general formulation based on the Boundary Element Method developed by the authors has allowed to identify this family of primitive methods as the result of introducing suitable assumptions in the BEM approach in order to reduce computational cost for specific choices of the test and trial functions. Furthermore, the anomalous asymptotic behaviour of this kind of methods could be mathematically explained, and sources of error have been pointed out[4], while more efficient and accurate formulations have been derived[5]. On the other hand, this BEM approach has been succesfully applied to the analysis of grounding systems in electrical substations, with a very reasonable computational cost in memory storage and CPU time[6].

Physical phenomena of fault currents dissipation into the earth can be modelled by means of Maxwell's Electromagnetic Theory[7]. Constraining the analysis to the obtention of the electrokinetic steady-state response and neglecting the inner resistivity of the earthing conductors —therefore, potential can be assumed constant in every point of the electrodes surface—, the 3D problem can be written as

$$\begin{gathered}\mathbf{div}\boldsymbol{\sigma} = 0, \qquad \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\mathbf{grad}V \ \text{ in } \ E; \\ \boldsymbol{\sigma}^t\boldsymbol{n}_E = 0 \ \text{ in } \ \Gamma_E; \quad V = V_\Gamma \ \text{ in } \ \Gamma; \quad V \longrightarrow 0, \ \text{ if } \ |\boldsymbol{x}| \to \infty;\end{gathered} \tag{1}$$

where E is the earth, $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the electrode surface[4,5]. Thus, when the electrode attains a voltage V_Γ (Ground Potential Rise or GPR) relative to a distant grounding point, the solution to this problem gives the potential V and the current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$. Since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, the normalized boundary condition $V_\Gamma = 1$ is not restrictive at all, and will be used from here on[5].

Furthermore, the grounding design parameters such as the leakage current density σ at an arbitrary point of the electrode surface, the total surge current I_Γ that flows into the ground during a fault condition, and the equivalent resistance of the earthing system R_{eq} (apparent resistance of the earth-electrode circuit) can be obtained as

$$\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}, \qquad I_\Gamma = \int\int_\Gamma \sigma \; d\Gamma, \qquad R_{eq} = \frac{V_\Gamma}{I_\Gamma}, \tag{2}$$

being $\boldsymbol{n}$ the normal exterior unit field to Γ.

Most of the methods proposed up to this moment are based on the assumption that the soil can be considered homogeneous and isotropic[8]. Thus, conductivity tensor $\boldsymbol{\gamma}$ is substituted by an apparent scalar conductivity γ that can be experimentally obtained[1]. As a general rule, this assumption does not introduce significant errors if the soil is essentially uniform (horizontally and vertically) up to a distance of approximately 3 to 5 times the diagonal dimension of the grid, measured from its edge. This uniform soil model can be also used with less accuracy if the resistivity varies slightly with depth[8]. Nevertheless, since parameters involved in the grounding design can significantly change as soil conductivity varies through the substation site, it seems reasonable to seek for more accurate models that could take into account the variation of soil conductivity in the surroundings of the grounding system.

At this point, the development of models describing all variations of the soil conductivity in the vicinity of an earthing system would rarely be affordable, from both economical and technical points of view. A more practical (and still quite realistic) approach to situations where conductivity is not markedly uniform with depth consists of considering the soil stratified in a number of horizontal layers, which appropriate thickness and apparent scalar conductivity must be experimentally obtained. In fact, it is widely accepted that two layer earth models should be sufficient to obtain good and safe designs of grounding systems in most practical cases[1,8].

When the grounding electrode is buried in the upper layer of the soil, the mathematical problem (1) can be rewritten as the Neumann Exterior Problem:

$$\begin{gathered} \Delta V_1 = 0 \quad \text{in} \quad E_1, \quad \Delta V_2 = 0 \quad \text{in} \quad E_2; \\ \frac{dV_1}{dn} = 0 \quad \text{in} \quad \Gamma_E, \quad V_1 = V_2 \quad \text{in} \quad \Gamma_L, \quad \gamma_1 \frac{dV_1}{dn} = \gamma_2 \frac{dV_2}{dn} \quad \text{in} \quad \Gamma_L, \\ V_1 = V_\Gamma \quad \text{in} \quad \Gamma, \quad V_1 \longrightarrow 0 \quad \text{y} \quad V_2 \longrightarrow 0 \quad \text{si} \quad |\boldsymbol{x}| \rightarrow \infty, \end{gathered} \tag{3}$$

where E_1 and E_2 are the upper and lower layers of the earth, Γ_L is the interface between them, γ_1 and γ_2 are the respective apparent scalar conductivities of both layers, and V_1 and V_2 are the corresponding expressions of the potential in each one of them[9,10]. A scheme of this situation is shown in figure 1. Obviously, if the grounding electrode is buried in the lower layer of the soil ($V_2 = V_\Gamma$ in Γ), the statement of the exterior problem is analogous to (3)[10].

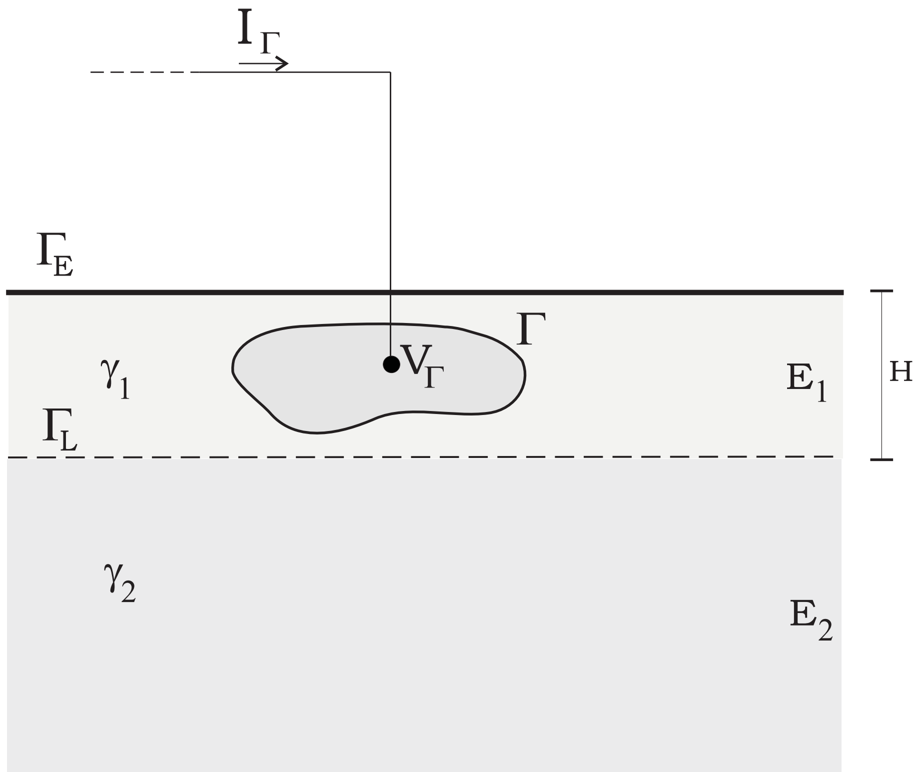

Figure 1.—Schematical representation of the fault current dissipation into the earth through a grounding electrode embedded in a two layer soil.

2. VARIATIONAL STATEMENT OF THE PROBLEM

In most of real electrical installations, the particular geometry of the grounding electrode —a grid of interconnected bare cylindrical conductors, horizontally buried and supplemented by a number of vertical rods, which ratio diameter/lenght uses to be relatively small ($\sim 10^{-3}$)— precludes the obtention of analytical solutions. On the other hand, the use of standard numerical techniques (such as Finite Differences or Finite Elements) requires the discretization of domains E_1 and E_2, and the obtention of sufficiently accurate results would imply an extremely high (out of range) computational effort.

At this point, we remark that computation of potential is only required on Γ_E, and the equivalent resistance can be easily obtained in terms of the leakage current density σ by means of (2). Therefore, we turn our attention to a Boundary Integral approach, which will only require the discretization of Γ, and will reduce the 3D problem to a 2D one.

Thus, if one further assumes that the earth surface Γ_E and the interface between the two soil layers Γ_L are horizontal, symmetry (method of images) allows to rewrite (3) in terms of a Dirichlet Exterior Problem[5,10]. This hypothesis of horizontal surfaces seems to be quite adequate, if we take into account that, in practice, surroundings of almost every electrical installation must be levelled before its construction.

The application of Green's Identity[6,10] to this Dirichlet Exterior Problem yields to the following integral expressions for potential $V_1(\boldsymbol{x}_1)$ and $V_2(\boldsymbol{x}_2)$, at arbitrary points $\boldsymbol{x}_1$ in E_1 and $\boldsymbol{x}_2$ in E_2, in terms of the unknown leakage current density $\sigma(\boldsymbol{\xi})$, at any point $\boldsymbol{\xi}$ —with coordinates $[\xi_x, \xi_y, \xi_z]$— on electrode surface Γ:

$$V_1(\boldsymbol{x}_1) = \frac{1}{4\pi\gamma_1} \int\int_{\boldsymbol{\xi}\in\Gamma} k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma, \quad \forall \boldsymbol{x}_1 \in E_1; \tag{4}$$

$$V_2(\boldsymbol{x}_2) = \frac{1}{4\pi\gamma_1} \int\int_{\boldsymbol{\xi}\in\Gamma} k_{12}(\boldsymbol{x}_2, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma, \quad \forall \boldsymbol{x}_2 \in E_2; \tag{5}$$

being $k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi})$ and $k_{12}(\boldsymbol{x}_2, \boldsymbol{\xi})$ the weakly singular kernels

$$\begin{aligned} k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi}) = {} & \frac{1}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, \xi_z])} + \frac{1}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -\xi_z])} \\ & + \sum_{i=1}^{\infty} \Big[\frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH + \xi_z])} + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH - \xi_z])} \\ & \quad + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH + \xi_z])} + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH - \xi_z])} \Big]; \end{aligned} \tag{6}$$

$$\begin{aligned} k_{12}(\boldsymbol{x}_2, \boldsymbol{\xi}) = {} & \frac{1+\kappa}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, \xi_z])} + \frac{1+\kappa}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, -\xi_z])} \\ & + \sum_{i=1}^{\infty} \Big[\frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, 2iH + \xi_z])} + \frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, 2iH - \xi_z])} \Big]; \end{aligned} \tag{7}$$

where $r(\boldsymbol{x}, [\xi_x, \xi_y, \xi_z])$ indicates the distance from $\boldsymbol{x}$ to $\boldsymbol{\xi} \equiv [\xi_x, \xi_y, \xi_z]$ —and to the symmetric points of $\boldsymbol{\xi}$ with respect to the earth surface Γ_E and the interface surface Γ_L between layers, which appear in the different terms in (6) and (7)—, H is the heigth of the upper soil layer, and κ is a relation between the conductivities of both layers[10,11],

$$\kappa = \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2}. \tag{8}$$

On the other hand, if the grounding electrode is buried in the lower layer of the earth, the application of Green's Identity[10] allows to obtain the following expressions —analogous to (4) and (5)— for potentials V_1 and V_2:

$$V_1(\boldsymbol{x}_1) = \frac{1}{4\pi\gamma_2} \int\int_{\boldsymbol{\xi}\in\Gamma} k_{21}(\boldsymbol{x}_1, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma, \quad \forall \boldsymbol{x}_1 \in E_1; \tag{4a}$$

$$V_2(\boldsymbol{x}_2) = \frac{1}{4\pi\gamma_2} \int\int_{\boldsymbol{\xi}\in\Gamma} k_{22}(\boldsymbol{x}_2, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma, \quad \forall \boldsymbol{x}_2 \in E_2; \tag{5a}$$

being $k_{21}(\boldsymbol{x}_1, \boldsymbol{\xi})$ and $k_{22}(\boldsymbol{x}_2, \boldsymbol{\xi})$ the weakly singular kernels

$$\begin{aligned} k_{21}(\boldsymbol{x}_1, \boldsymbol{\xi}) = & \frac{1-\kappa}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, \xi_z])} + \frac{1-\kappa}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -\xi_z])} \\ & + \sum_{i=1}^{\infty} \Big[\frac{(1-\kappa)\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH + \xi_z])} + \frac{(1-\kappa)\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH - \xi_z])} \Big]; \end{aligned} \tag{6a}$$

$$\begin{aligned} k_{22}(\boldsymbol{x}_2, \boldsymbol{\xi}) = & \frac{1}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, \xi_z])} + \frac{1-\kappa^2}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, -\xi_z])} \\ & + \frac{-\kappa}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, 2H + \xi_z])} + \sum_{i=1}^{\infty} \frac{(1-\kappa^2)\kappa^i}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, -2iH + \xi_z])}. \end{aligned} \tag{7a}$$

As it can be shown, integral expressions for the potential in the two possible situations of the grounding electrode —(4), (5), (4a) and (5a)— are essentially the same, and the differences in analytical expressions of integral kernels —(6), (7), (6a) and (7a)— are owed to the application of the method of images for each one of the cases[9,10,11]. Since the variational statement of the problem and the derivation of the numerical formulation are analogous in both cases, further development in this paper is restricted to the situation in which earthing electrode is buried in the upper layer.

In this case, since (3) holds on the earthing electrode surface Γ and the potential is known by the boundary condition on the Ground Potential Rise ($V(\boldsymbol{\chi}) = 1$, $\boldsymbol{\chi} \in \Gamma$), the leakage current density σ must satisfy the Fredholm integral equation of the first kind defined on Γ

$$1 = \frac{1}{4\pi\gamma_1} \int\int_{\boldsymbol{\xi}\in\Gamma} k_{11}(\boldsymbol{\chi}, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi})\; d\Gamma, \quad \boldsymbol{\chi} \in \Gamma. \tag{9}$$

Finally, a weaker variational form[6] of equation (9) can now be written as:

$$\iint_{\boldsymbol{\chi}\in\Gamma} w(\boldsymbol{\chi}) \left(\frac{1}{4\pi\gamma_1} \iint_{\boldsymbol{\xi}\in\Gamma} k_{11}(\boldsymbol{\chi}, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi})\; d\Gamma - 1 \right) d\Gamma \;=\; 0, \tag{10}$$

which must hold for all members $w(\boldsymbol{\chi})$ of a suitable class of test fuctions defined on Γ.

Obviously, a Boundary Element formulation seems to be the right choice to solve variational statement (10).

3. BOUNDARY ELEMENT FORMULATION

For a given set of $\mathcal{N}$ trial functions $\{N_i(\boldsymbol{\xi})\}$ defined on Γ, and for a given set of $\mathcal{M}$ 2D boundary elements $\{\Gamma^\alpha\}$, the unknown leakage current density σ and the grounding electrode surface Γ can be discretized in the form,

$$\sigma(\boldsymbol{\xi}) = \sum_{i=1}^{\mathcal{N}} \sigma_i \, N_i(\boldsymbol{\xi}), \qquad \Gamma = \bigcup_{\alpha=1}^{\mathcal{M}} \Gamma^\alpha, \tag{11}$$

and expressions (4) and (5) can be approximated as

$$V_1(\boldsymbol{x}_1) = \sum_{i=1}^{\mathcal{N}} \sigma_i \, V_{1i}(\boldsymbol{x}_1), \qquad V_{1i}(\boldsymbol{x}_1) = \sum_{\alpha=1}^{\mathcal{M}} V_{1i}^\alpha(\boldsymbol{x}_1), \quad \forall \boldsymbol{x}_1 \in E_1; \tag{12}$$

$$V_2(\boldsymbol{x}_2) = \sum_{i=1}^{\mathcal{N}} \sigma_i \, V_{2i}(\boldsymbol{x}_2), \qquad V_{2i}(\boldsymbol{x}_2) = \sum_{\alpha=1}^{\mathcal{M}} V_{2i}^\alpha(\boldsymbol{x}_2), \quad \forall \boldsymbol{x}_2 \in E_2; \tag{13}$$

being potential coefficients V_{1i}^α and V_{2i}^α,

$$V_{1i}^\alpha(\boldsymbol{x}_1) = \frac{1}{4\pi\gamma_1} \int\!\!\int_{\boldsymbol{\xi}\in\Gamma^\alpha} k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi}) \, N_i(\boldsymbol{\xi}) \; d\Gamma^\alpha, \quad \forall \boldsymbol{x}_1 \in E_1; \tag{14}$$

$$V_{2i}^\alpha(\boldsymbol{x}_2) = \frac{1}{4\pi\gamma_1} \int\!\!\int_{\boldsymbol{\xi}\in\Gamma^\alpha} k_{12}(\boldsymbol{x}_2, \boldsymbol{\xi}) \, N_i(\boldsymbol{\xi}) \; d\Gamma^\alpha, \quad \forall \boldsymbol{x}_2 \in E_2. \tag{15}$$

Moreover, for a given set of $\mathcal{N}$ test functions $\{w_j(\boldsymbol{\chi})\}$ defined on Γ, the variational statement (10) is reduced to the system of linear equations

$$\sum_{i=1}^{\mathcal{N}} R_{ji}\sigma_i = \nu_j, \qquad j = 1, \ldots, \mathcal{N}; \tag{16}$$

$$R_{ji} = \sum_{\beta=1}^{\mathcal{M}} \sum_{\alpha=1}^{\mathcal{M}} R_{ji}^{\beta\alpha}, \qquad \nu_j = \sum_{\beta=1}^{\mathcal{M}} \nu_j^\beta; \tag{17}$$

$$R_{ji}^{\beta\alpha} = \frac{1}{4\pi\gamma_1} \int\!\!\int_{\boldsymbol{\chi}\in\Gamma^\beta} w_j(\boldsymbol{\chi}) \int\!\!\int_{\boldsymbol{\xi}\in\Gamma^\alpha} k_{11}(\boldsymbol{\chi}, \boldsymbol{\xi}) \, N_i(\boldsymbol{\xi}) \, d\Gamma^\alpha d\Gamma^\beta \tag{18}$$

$$\nu_j^\beta = \int\!\!\int_{\boldsymbol{\chi}\in\Gamma^\beta} w_j(\boldsymbol{\chi}) \; d\Gamma^\beta. \tag{19}$$

It is important to remark that expression (18) is satisfied when all electrodes of the grounding grid are buried in the upper layer. Obviously, if a part of the earthing grid is in the lower layer ($\boldsymbol{\chi} \in E_2$), the integral kernel in (18) must be substituted[10] by $k_{12}(\boldsymbol{\chi}, \boldsymbol{\xi})$.

In practice, the 2D discretization required to solve the above stated equations in real problems implies an extremely large number of degrees of freedom. In addition, if we take into account that the coefficient matrix in (16) is full and the computation of each contribution (18) requires an extremely high number of evaluations of the kernel and double integration on a 2D domain[10], it is necessary to introduce some additional simplifications in the BEM approach to decrease the computational cost[6].

4. APPROXIMATED 1D BOUNDARY ELEMENT FORMULATION

With this aim, and considering the real geometry of grounding systems in most of substations, one can assume that the leakage current density is constant around the cross section of the cylindrical electrode[4]. This hypothesis of circumferential uniformity is widely used in most of the theoretical developments and practical techniques related in the literature[1,8].

Thus, let L be the whole set of axial lines of the buried conductors, $\widehat{\boldsymbol{\xi}}$ the orthogonal projection over the bar axis of a given generic point $\boldsymbol{\xi} \in \Gamma$, $\phi(\widehat{\boldsymbol{\xi}})$ the electrode diameter, and $\widehat{\sigma}(\widehat{\boldsymbol{\xi}})$ the approximated leakage current density at this point (assumed uniform around the cross section). In these terms, and being $\bar{k}_{11}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ and $\bar{k}_{12}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ the average of kernels (6) and (7) around the cross section at $\widehat{\boldsymbol{\xi}}$, we can obtain[6] the approximated expressions of potential (4) and (5) as,

$$\widehat{V}_1(\boldsymbol{x}_1) = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}_{11}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\; dL, \quad \forall \boldsymbol{x}_1 \in E_1; \tag{20}$$

$$\widehat{V}_2(\boldsymbol{x}_2) = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}_{12}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\; dL, \quad \forall \boldsymbol{x}_2 \in E_2. \tag{21}$$

Hereby, since the leakage current is not exactly uniform around the cross section, boundary condition $V_1(\boldsymbol{\chi}) = 1$, $\boldsymbol{\chi} \in \Gamma$ will not be strictly satisfied at every point $\boldsymbol{\chi}$ on the electrode surface Γ, and variational equality (10) will not hold anymore. However, if we restrict the class of trial functions to those with circumferential uniformity, (10) results in

$$\frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}}) \left[\int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL \right] dL = \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}})\, dL, \tag{22}$$

which must hold for all members $\widehat{w}(\widehat{\boldsymbol{\chi}})$ of a suitable class of test functions defined on L, being $\bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ the average of kernel $k_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ in (6) around the cross sections at $\widehat{\boldsymbol{\chi}}$ and $\widehat{\boldsymbol{\xi}}$[6].

Resolution of integral equation (22) involves discretization of the domain formed by the whole set of axial lines of the buried conductors L. Thus, for given sets of n

trial functions $\{\widehat{N}_i(\widehat{\boldsymbol{\xi}})\}$ defined on L, and m 1D boundary elements $\{L^\alpha\}$, the unknown approximated leakage current density $\widehat{\sigma}$ and the set of axial lines L can be discretized in the form

$$\widehat{\sigma}(\widehat{\boldsymbol{\xi}}) = \sum_{i=1}^{n} \widehat{\sigma}_i\, \widehat{N}_i(\widehat{\boldsymbol{\xi}}), \qquad L = \bigcup_{\alpha=1}^{m} L^\alpha, \tag{23}$$

and a discretized version of approximated potential (20) and (21) can be obtained as

$$\widehat{V}_1(\boldsymbol{x}_1) = \sum_{i=1}^{\mathcal{N}} \widehat{\sigma}_i\, \widehat{V}_{1i}(\boldsymbol{x}_1), \qquad \widehat{V}_{1i}(\boldsymbol{x}_1) = \sum_{\alpha=1}^{\mathcal{M}} \widehat{V}_{1i}^{\alpha}(\boldsymbol{x}_1), \quad \forall \boldsymbol{x}_1 \in E_1; \tag{24}$$

$$\widehat{V}_2(\boldsymbol{x}_2) = \sum_{i=1}^{\mathcal{N}} \widehat{\sigma}_i\, \widehat{V}_{2i}(\boldsymbol{x}_2), \qquad \widehat{V}_{2i}(\boldsymbol{x}_2) = \sum_{\alpha=1}^{\mathcal{M}} \widehat{V}_{2i}^{\alpha}(\boldsymbol{x}_2), \quad \forall \boldsymbol{x}_2 \in E_2; \tag{25}$$

being potential coefficients $\widehat{V}_{1i}^{\alpha}$ and $\widehat{V}_{2i}^{\alpha}$,

$$\widehat{V}_{1i}^{\alpha}(\boldsymbol{x}_1) = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}}\in L^\alpha} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}_{11}(\boldsymbol{x}_1, \widehat{\boldsymbol{\xi}})\, \widehat{N}_i(\widehat{\boldsymbol{\xi}})\; dL^\alpha, \quad \forall \boldsymbol{x}_1 \in E_1; \tag{26}$$

$$\widehat{V}_{2i}^{\alpha}(\boldsymbol{x}_2) = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}}\in L^\alpha} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}_{12}(\boldsymbol{x}_1, \widehat{\boldsymbol{\xi}})\, \widehat{N}_i(\widehat{\boldsymbol{\xi}})\; dL^\alpha, \quad \forall \boldsymbol{x}_2 \in E_2. \tag{27}$$

Finally, for a suitable selection of n test functions $\{\widehat{w}_j(\widehat{\boldsymbol{\chi}})\}$ defined on L, equation (22) is reduced to the system of linear equations[6,10],

$$\sum_{i=1}^{n} \widehat{R}_{ji}\widehat{\sigma}_i = \widehat{\nu}_j, \qquad j = 1, \ldots, n; \tag{28}$$

$$\widehat{R}_{ji} = \sum_{\beta=1}^{m}\sum_{\alpha=1}^{m} \widehat{R}_{ji}^{\beta\alpha}, \qquad \widehat{\nu}_j = \sum_{\beta=1}^{m} \widehat{\nu}_j^{\beta}; \tag{29}$$

$$\widehat{R}_{ji}^{\beta\alpha} = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\chi}}\in L^\beta} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}_j(\widehat{\boldsymbol{\chi}}) \int_{\widehat{\boldsymbol{\xi}}\in L^\alpha} \phi(\widehat{\boldsymbol{\xi}})\, \bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})\, \widehat{N}_i(\widehat{\boldsymbol{\xi}})\, dL^\alpha dL^\beta \tag{30}$$

$$\widehat{\nu}_j^{\beta} = \int_{\widehat{\boldsymbol{\chi}}\in L^\beta} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}_j(\widehat{\boldsymbol{\chi}})\, dL^\beta. \tag{31}$$

Just like the previous 2D approach, we remark that expression (30) is satisfied when all electrodes of the grounding grid are buried in the upper layer. Obviously, if a part of the earthing grid is in the lower layer ($\widehat{\boldsymbol{\chi}} \in E_2$), the integral kernel in (30) must be substituted[10] by $\bar{\bar{k}}_{12}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$.

The matrix of coefficients of this approximated 1D problem is still full. However, on a regular basis we can say that the computational cost has been drastically reduced, since the actual discretization (1D) in (28) for a given problem will be much simpler than before (2D) in (16). Furthermore, suitable unexpensive approximations[5] can be introduced to evaluate the averaged kernels $\bar{k}_{11}(\boldsymbol{x},\widehat{\boldsymbol{\xi}})$, $\bar{k}_{12}(\boldsymbol{x},\widehat{\boldsymbol{\xi}})$, $\bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}},\widehat{\boldsymbol{\xi}})$ and $\bar{\bar{k}}_{12}(\widehat{\boldsymbol{\chi}},\widehat{\boldsymbol{\xi}})$.

On the other hand, computation of line integrals involved in (30) is not obvious, and standard numerical integration cannot be used due to the undesirable behaviour of the integrands. Nevertheless, suitable arrangements in the final expressions of the matrix coefficients can be performed, so that it is possible to use the highly efficient analytical integration techniques derived by the authors in the last years in cases of grounding systems in uniform soils[5,6].

This BEM approach has been implemented in the CAD system for grounding grids of electrical installations recently developed by the authors[12,13,14]. It is important to notice that the total computing effort required in some cases is very high, particularly in those in which conductivities of soil layers are very different ($|\kappa| \approx 1$). The fact is that the rate of convergence of averaged kernels $\bar{k}_{11}(\cdot,\cdot)$, $\bar{k}_{12}(\cdot,\cdot)$, $\bar{\bar{k}}_{11}(\cdot,\cdot)$ and $\bar{\bar{k}}_{12}(\cdot,\cdot)$ —which are very similar to (6) and (7)— is very low when $|\kappa| \approx 1$, and an extremely large number of terms is necessary to compute in order to obtain accurate results[10,14].

5. APPLICATION TO REAL CASES

This formulation has been applied to two real cases. The first example we present is the E.R.Barberá substation grounding operated by the power company Fecsa, close to the city of Barcelona in Spain. The earthing system of this substation is a grid of 408 cylindrical conductors with constant diameter (12.85 mm) buried to a depth of 80 cm, being the total surface protected up to 6500 m^2. The total area studied is a rectangle of 135 m by 210 m, which implies a surface up to 28000 m^2. The Ground Potential Rise considered in this study is 10 kV.

Table I.—E. R. Barberá Substation: Characteristics and Numerical Model

Data		1D BEM Model	
Number of Electrodes:	408	Type of Approach:	Galerkin
Electrode Diameter:	12.85 mm	Type of Element:	Linear
Installation Depth:	0.8 m	Number of Elements:	408
Max. Grid Dimensions:	145 m × 90 m	Degrees of Freedom:	238
Ground Potential Rise:	10 kV		

The numerical model used in this problem is based on a Galerkin type weighting. Thus, each bar is discretized in one single linear leakage current density element, which implies a total of 238 degrees of freedom. The characteristics, numerical model and plan of the grounding grid are presented in table I and figure 2.

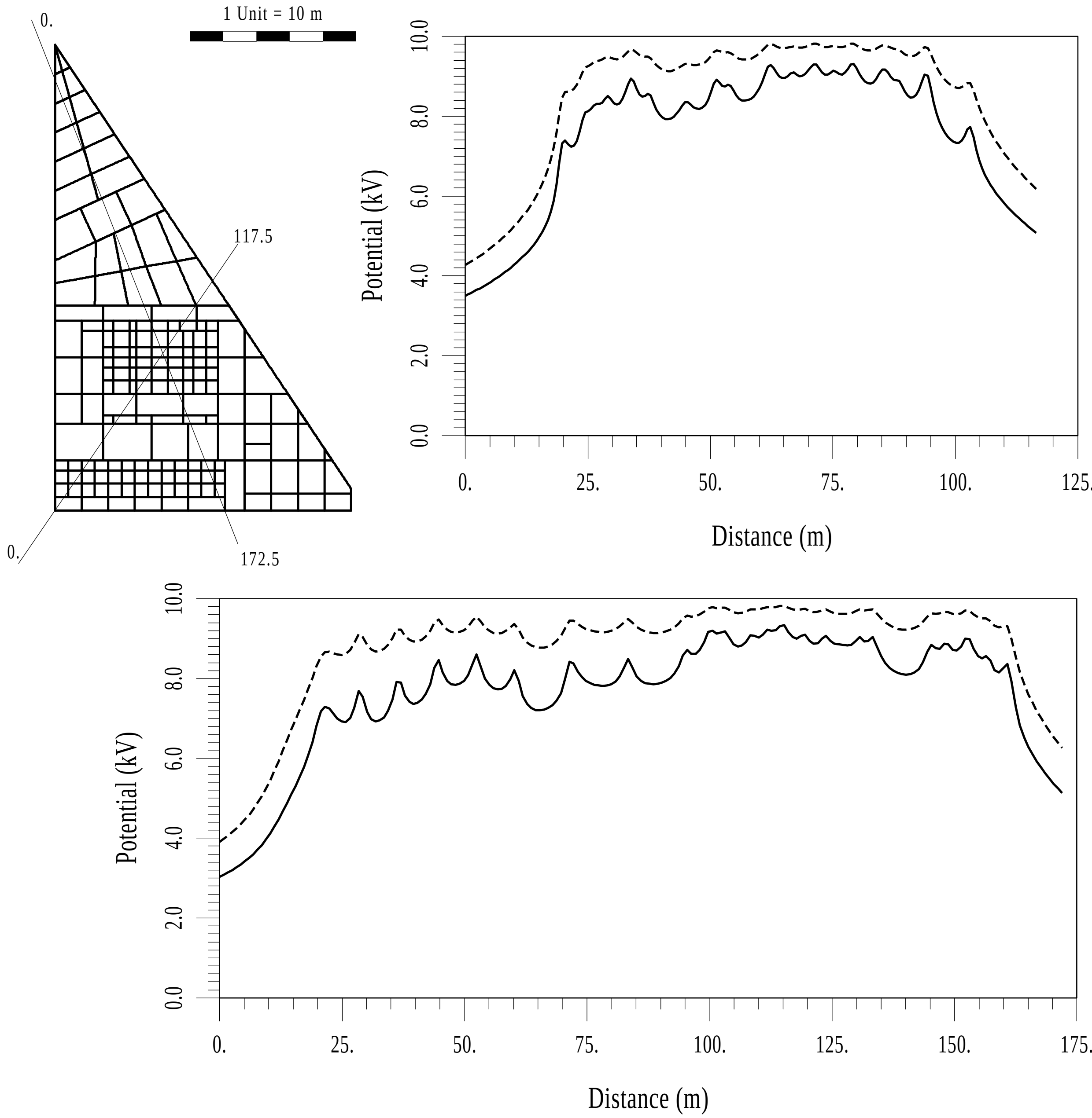

Figure 2.—E. R. Barberá Substation: Plan of the Grounding Grid and Potential profiles along two different lines (results obtained by using an uniform soil model are indicated with discontinuous line, and those obtained by using a two layer model are given with continuous line).

The equivalent resistance, the fault current and potential profiles along different lines obtained with this BEM approach by using a two layer soil model are compared with those obtained by using an uniform soil model (figure 2 and table II).

The second example presented in this paper is the Balaidos II substation grounding

Table II.—E. R. Barberá Substation: Results by using different soil models

Two Layer Soil Model		**Uniform Soil Model**	
Upper Layer Resistivity :	200 Ω m	—	
Lower Layer Resistivity :	60 Ω m	—	
Height of Upper Layer :	1.2 m	Earth Resistivity :	60 Ω m
Fault Current :	25.88 kV	Fault Current :	31.85 kV
Equivalent Resistance :	0.386 Ω	Equivalent Resistance :	0.314 Ω
CPU Time (AXP 4000):	5.9 min.	CPU Time (AXP 4000):	3.9 sec.

operated by the power company Unión Fenosa, close to the city of Vigo in Spain. The earthing system of this substation is a grid of 107 cylindrical conductors (diameter: 11.28 mm) buried to a depth of 80 cm, supplemented with 67 vertical rods (each one has a length of 2.5 m and a diameter of 14.0 mm). The total surface protected up to 4800 m^2. The total area studied is a rectangle of 121 m by 108 m, which means a surface up to 13000 m^2. As in the previous case, the Ground Potential Rise considered in this study has been 10 kV.

Table III.—E.R. Balaídos II Substation: Characteristics and Numerical Model

Data		**1D BEM Model**	
Number of Electrodes :	107	Type of Approach :	Galerkin
Number of Vertical Rods :	67	Type of Element :	Linear
Electrode Diameter :	11.28 mm	Number of Elements :	241
Vertical Rod Diameter :	14.00 mm	Degrees of Freedom :	208
Installation Depth :	0.8 m		
Vertical Rod Length :	2.5 m		
Max. Grid Dimensions :	60 m $\times$ 80 m		
Ground Potential Rise :	10 kV		

The plan of the grounding grid is presented in figure 3, and its general characteristics and the numerical model used are summarized in table III.

Results, such as the equivalent resistance, total surge current and potential profiles on the earth surface along two different lines obtained with the BEM formulation by using a two layer soil model are compared with those obtained by using an uniform soil model (figure 4 and table IV).

In figure 5, we represent the potential distribution (kV) into the ground along a line on the earth surface obtained by using the uniform soil model —figure 5a)— and the

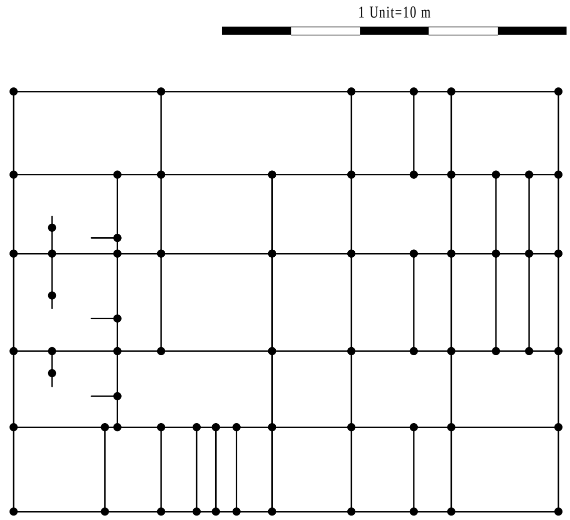

Figure 3.—E.R. Balaídos II Substation: Plan of the Grounding Grid (Vertical rods marked with black points).

Table IV.—E.R. Balaídos II Substation: Results by using different soil models

Two Layer Soil Model		**Uniform Soil Model**	
Upper Layer Resistivity :	200 Ω m	—	
Lower Layer Resistivity :	60 Ω m	—	
Height of Upper Layer :	1.2 m	Earth Resistivity :	60 Ω m
Fault Current :	21.14 kA	Fault Current :	24.94 kA
Equivalent Resistance :	0.473 Ω	Equivalent Resistance :	0.401 Ω
CPU Time (AXP 4000):	37.5 sec.	CPU Time (AXP 4000):	1.5 sec.

two layer soil model —figure 5b)—. The contour lines in a more detailed zone of figures a) and b) obtained by means of the uniform soil model and the two layer are given in figures 5c) and 5d).

We remark that the analysis of this grounding system with the two layer soil model is particularly difficult because the length of the vertical rods (2.5 m) are higher than the height of the upper layer (1.2 m), and in consequence, a part of the grid is buried in the upper layer and other part in the lower. In cases like this, the final implementation of the numerical approach in a computer aided design system must be done with care, in order to combine properly the different expressions that we obtain with the analysis of the possible situation of electrodes.

It can be shown in these examples that results obtained by using different soil models are noticeably different and accordingly, the design parameters of a grounding system[5] (such as the equivalent resistance, the touch voltage, the step voltage, the mesh voltage,

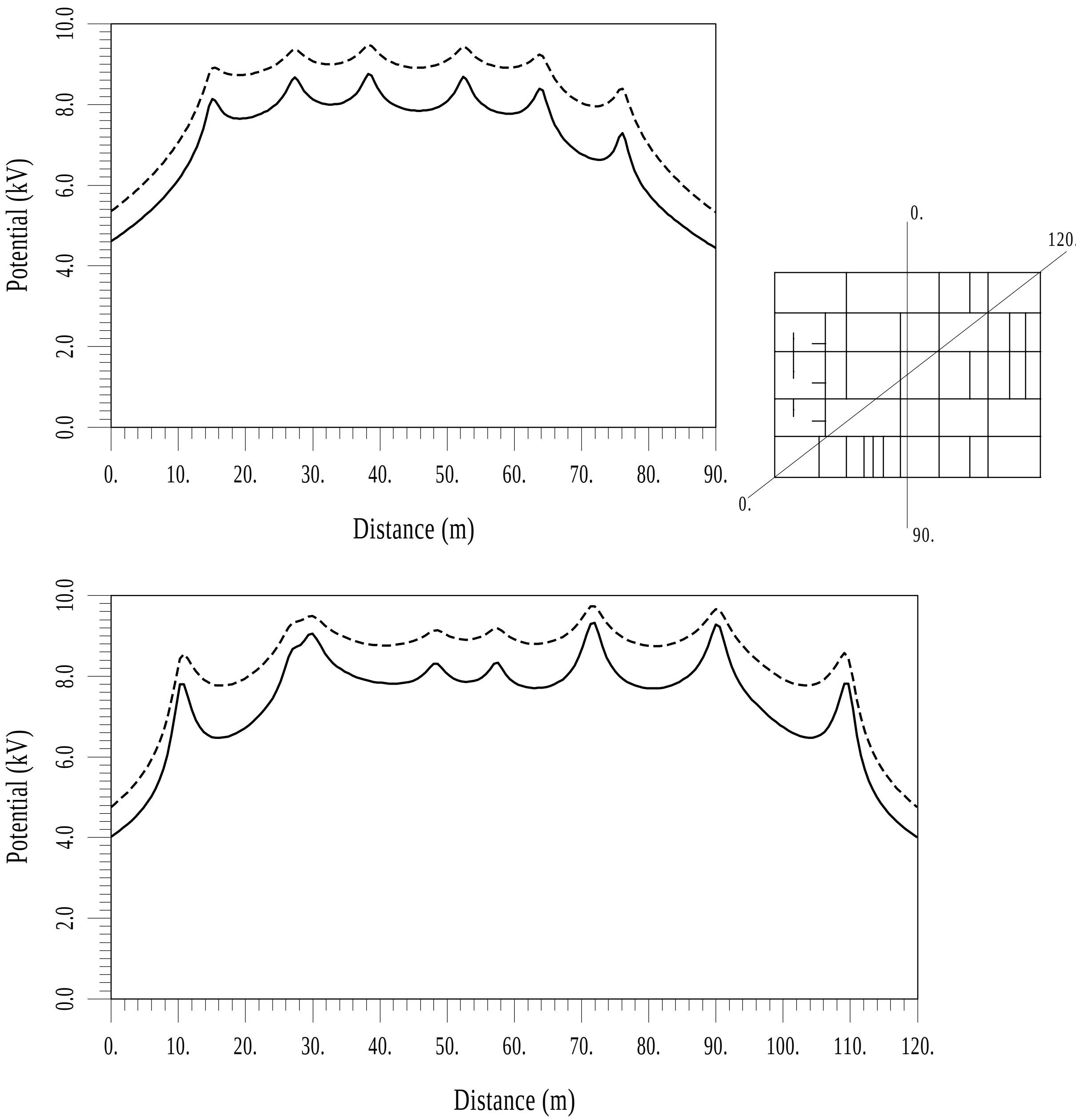

Figure 4.—E.R. Balaídos II : Potential profiles along two different lines (results obtained by using an uniform soil model are indicated with discontinuous line, and those obtained by using a two layer model are given with continuous line).

etc.) may significantly vary. Therefore, in spite of the increase in the computational effort it will be essential to analyze grounding systems with this new BEM technique, in cases where the conductivity of the soil changes markedly with depth and so the hypothesis of uniform soil model is not valid.

6. CONCLUSIONS

A Boundary Element formulation for the analysis of substation grounding systems embedded in layered soils has been presented. This approach has been applied to the practical case of an earthing system in an equivalent two layer soil.

According to the specific characteristics of these installations in practice, some reasonable assumptions allow to reduce a general 2D BEM approach to an approximated 1D version. Furthermore, suitable arrangements in the final discretized equations can be performed in such a way that it is possible to use the highly efficient analytical integration techniques that have been derived by the authors in cases of grounding systems buried in uniform soils[6].

This BEM technique has been implemented in the Computer Aided Design system developed by the authors for the grounding substation design[5]. With this system, it is possible to obtain highly accurate results in real problems. However, at present the study of larger installations still requires an important computing effort due to the large number of terms of integral kernels that it is necessary to evaluate. The application of new extrapollation techniques that are being derived by the authors at the moment, will allow to accelerate their rate of convergence and reduce the actual computational cost.

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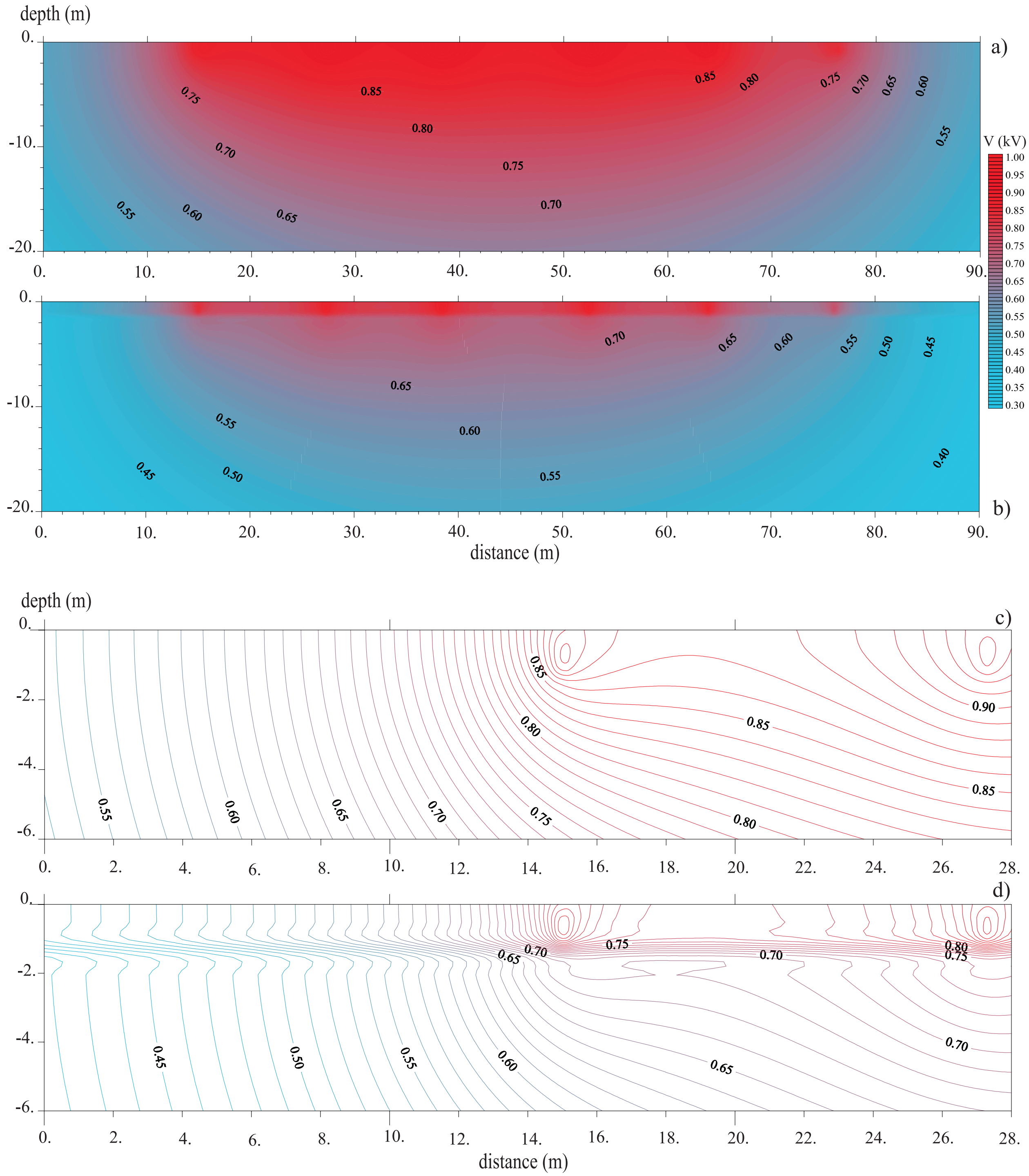

Figure 5.—E.R. Balaídos II : Potential distribution (kV) into the ground along the line of 90. m indicated in figure 4 obtained by using: a) the uniform soil model and b) the two layer soil model, and Contour lines in a particular zone obtained by using: c) the uniform and d) the two layer soil model.